W2L2 - APPLICATIONS WITH SEPARABLE EQUATIONS

GROWTH & DECAY

MANY NATURAL GROWTH & DECAY PROBLEMS FOLLOW:

$$\frac{dx}{dt} = kx \quad , \; k = \text{CONSTANT} \quad \leftarrow \text{Exponential}$$

Rate of growth/decay is proportional to "amount" present (x)

EX

A town had 25000 in 1970 & 30000 in 1980 if pop. grows exponentially, find pop. in 2010.

$$\frac{dP}{dt} = kP \Rightarrow \frac{1}{P}\,dP = k\,dt$$

$$\int \frac{1}{P}\,dP = \int k\,dt$$

$$\ln|P| = kt + C_1$$

$$|P| = e^{kt + C_1}$$

$$|P| = e^{C_1} \cdot e^{kt}$$

$$P = \pm e^{C_1} e^{kt}$$

$$P(t) = Ce^{kt}$$

$$P(0) = P_0$$

$$P_0 = Ce^{k(0)} \Rightarrow P_0 = C$$

$$P(t) = P_0 e^{kt}$$

← if k is pos → exponential growth
if k is neg → exponential decay

$$P_0 = 25000$$

$$P(10) = 30000$$

$$10000 = 25000\,e^{k(10)}$$

$$1.2 = e^{10k}$$

$$\ln(1.2) = 10k$$

$$k = \frac{\ln(1.2)}{10} \approx 0.01823$$

$$P(t) = 25000\,e^{0.018236}$$

2010 → $t = 40$ years

$$P(40) = 25000\,e^{0.01823(40)}$$

51840 = pop. in 2010

EX

A pop. of bunnies increased 6 fold in 10 months. How long did it take for the population to double?

$P(t) = P_0 e^{kt}$

$P_0 = 1$

$P(10) = 6$

$6 = 1e^{k(10)}$

$\ln|6| = 10k$

$k = \ln 6 / 10 \Rightarrow k \approx 0.179$

$P(t) = P_0 e^{0.179t}$

$2 = 1e^{0.179t}$

$\ln|2| = 0.179t$

$t = \frac{\ln|2|}{0.179} \Rightarrow t \approx 3.87$ months